7 Are Thinking Computers Mathematically Possible?

The History and Status of the Debate — Map 7 of 7

An Issue Map™ Publication

Start Here

1 Alan Turing, 1950
Yes, machines can (or will be able to) think. A computational system can possess all important elements of human thinking or understanding.

I believe that at the end of the century ... one will be able to speak of machines thinking without expecting to be contradicted.

Alan Turing

is supported by

2
Computer thought is mathematically possible. It is mathematically possible for a computer to think as well as a human can. The mathematics of computation contains nothing to prohibit machines from thinking.

There's no mathematical reason why computers can't think.

The Background of Gödel's Proof

Context
By the turn of the 20th century a crisis had developed in the foundations of mathematics. The discovery of fundamental paradoxes led to concern about the philosophical basis of math and logic. In response to these concerns, mathematicians tried to develop more secure foundational systems. (See sidebar, "Formal Systems: An Overview," on this map).

One such system, the *Principia Mathematica* of Bertrand Russell and Alfred North Whitehead, was widely received and provided a framework for subsequent work on the foundations of arithmetic, geometry, analysis, and algebra. Concurrently with Russell and Whitehead, David Hilbert worked on foundational mathematics. Hilbert's central idea was that the consistency of mathematics could be established by a system of metamathematics — a system of mathematics about mathematics.

The work of Kurt Gödel was situated in this context. He set out to prove the recently developed methods to his own areas of interest, and shortly thereafter discovered the proof for his famous incompleteness theorems.

Method
Gödel developed a numbering system that allowed him to encode formal metamathematical proofs into numeric expressions. The numeric expressions, or Gödel numerals, could be recognized and manipulated within the *Principia* system in the same way that any numeric expression could be (see sidebar, "The Steps of Gödel's Proof," on this map). Gödel numerals allowed the *Principia*—and formal systems generally—to "look at themselves" and say things about themselves. Through this method, Gödel wanted to find out whether the formal system of *Principia Mathematica* could prove itself consistent.

He discovered that no such proof exists. There is no way for the *Principia* to prove itself consistent. Gödel went further and used his method to prove that no complete formalization of arithmetic exists at all (see "Gödel's First Theorem," Box 4). As a corollary, he returned to the consistency question and showed that no consistent formal system of arithmetic could be proved consistent using only its own methods of proof (see "Gödel's Second Theorem," Box 5).

Earlier Precedents
Gödel wasn't the first to suspect that his completeness and consistency results held. Earlier, Finsler (1926) presented an idea similar to Gödel's but without showing how to formalize the argument. Because he did not deal with a specific formal system, Finsler could not present any actual proof of his claim. The American mathematician Ernst Post had also proved a result equivalent to Gödel's, but his work wasn't published.

Suggested Reading
Gödel's original paper in English contains the proofs, plus a lucid discussion of the meaning of the theorems, in Hamel (1987). For historical context, see Rucker (1987), Davis (1965), Dawson (1984a, 1984b), and van Heijenoort (1967). This last volume reprints Gödel's original paper.

The Steps of Gödel's Proof

1 The *Principia Mathematica* gave Gödel a way to translate natural language statements into a formal system of proof. With this formal system in hand, Gödel was able prove that all such statements, including those that mathematics are inherently limited. Illustration of the proof begins with an English version of what will become the notorious "Gödel sentence."

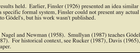
Sentence in natural language

"G says it is not provable that G."

2 In step 2, the sentence is roughly translated into the formal system of *Principia Mathematica*. The formalization shown here is not well-formed (i.e., is not "grammatically correct"), and consequently the system of proof used in *Principia Mathematica* cannot deal with it. Later, this sentence will be rendered well-formed and formal mustfirst develop a way to make well formed statements that refer to themselves.

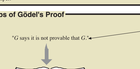
Formalization of Gödel sentence in the formal system (e.g., *Principia Mathematica*)

3 To that end, each symbol in the formal system is replaced with a code number.



Code numbers

4 These numbers are then used as exponents in a series of prime numbers that will be multiplied together into one large *Gödel number*. This step ensures that every Gödel number corresponds to one and only one formula in the formal system of *Principia Mathematica*.



Gödel number

5 This number is then represented symbolically, in this case, by a long string of S's followed by a 0 (where the number of S's is equal to the Gödel number). The symbolic "numeral" representation is necessary so that the Gödel number can be dealt with formally in *Principia Mathematica*.

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